

2.1 Derivative Basics: Form E Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_set.py`). All problems are original; see the notes file for provenance. Prepared with assistance from Claude, an AI model made by Anthropic.

General grading rules

- **Answer-only items** (Q1, Q9, Q13): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse) earns 0 for that part.
- **Show-work items** (Parts II–IV other than Q9): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form E

Item	Answer	Rubric / trap
1(a)	-1	<i>Trap:</i> the tangent drawn there invites the slope, -2 ; earns 0.
1(b)	2	<i>Trap:</i> giving the height $f(-3) = -1$; earns 0.
1(c)	0	<i>Trap:</i> giving the height -2 of the low point; earns 0.
1(d)	DNE	Corner at $(-2, 2)$: one-sided slopes 4 and 2. <i>Trap:</i> a slope of 2 or 4, or deciding the derivative exists because $(-2, 2)$ is not a high or low point.
1(e)	$(-4, -2)$ and $(-2, 3)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-4, 3)$, which includes $x = -2$, where f' does not exist.
1(f)	$f'(-5) < 0 < f'(-1)$	$f'(-5) = -2$, $f'(-1) = 2$. 3 pts, all or nothing.
2	$y - 5 = -3(x + 2)$, i.e. $y = -3x - 1$	2 pts point $(-2, 5)$ and slope -3 in point-slope form; 3 pts equation. <i>Trap:</i> there is no formula to differentiate, and students hunt for one. Writing $x - 2$ for $x - (-2)$ earns 2.
3	$y + 3 = -6(x + 1)$, i.e. $y = -6x - 9$	1 pt $r(-1) = -3$; 2 pts $r'(x)$; 1 pt $r'(-1) = -6$; 1 pt equation in any correct form. <i>Trap:</i> $r'(x) = -4x^{-2} + 2x$, so $r'(-1) = -4 - 2 = -6$. $(-1)^{-2} = +1$; using -1 gives slope 2, which earns 3 of 5.
4(a)	1	3 pts. Simplifies to $y = x + 7$. <i>Trap:</i> 4, the coefficient in the familiar slope position; earns 0.
4(b)	5	2 pts. Substitute $x = -2$. <i>Trap:</i> 1, read from the left side; earns 0.
5	0	<i>Trap:</i> m' is printed and invites $m'(2) = 8$; earns 0.
6	$f'(x) = \frac{4}{(x+3)^2}$	2 pts $f(x+h) = -\frac{4}{x+h+3}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $-4(x+3) + 4(x+h+3) = 4h$; 2 pts cancel h to get $\frac{4}{(x+h+3)(x+3)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x)+h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	-5	1 pt $g(-1) = 2$; 1 pt setup; 3 pts factoring out $x + 1$ to get $x^3 - x^2 + x - 2$, by any valid method. Synthetic or long division by $x + 1$ works (on 1, 0, 0, -1, -2, with two zero placeholders). So does grouping: $x^4 - x - 2 = (x^4 - 1) - (x + 1) = (x + 1)(x - 1)(x^2 + 1) - (x + 1) = (x + 1)(x^3 - x^2 + x - 2)$. No particular method is required. 3 pts value. <i>Trap</i> : the denominator is $x + 1$; a student who divides and drops a placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	-1	$\sin(\pi + h) = \sin(\pi + h) - \sin \pi$, so this is the derivative of $\sin$ at π : $\cos \pi = -1$. <i>Trap</i> : 1 (the familiar $\frac{\sin h}{h}$) or "0/0, DNE"; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
8(b)	$-\frac{1}{16}$	the derivative of $\frac{1}{x}$ at 4: $-\frac{1}{16}$. <i>Trap</i> : $\frac{1}{16}$ (sign), or "0/0"; earns 0. 3 pts for recognizing which derivative the limit is (or a correct algebraic setup), 3 pts for the value.
9	(a) 1 (b) 0	3 each. <i>Traps</i> : (a) 0 (this one is not a constant: $a(x) = x$), or $x \cos(\pi/2)$; (b) $f'(-1) = -5$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says "for every x ".
10(a)	$12x^{-5} + \cos x$	1 pt rewriting $-3x^{-4}$; 2 pts derivative. <i>Trap</i> : $-3 \cdot (-4) = +12$; the exponent -5.
10(b)	$12x^2$	1 pt rewriting $4x^3 - 1$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : $\frac{8x^6}{2x^3} = 4x^3$, not $4x^2$; differentiating top and bottom separately earns 0.
10(c)	$12x^{1/3} - x^{-1/2}$	1 pt rewriting $9x^{4/3} - 2x^{1/2}$; 2 pts derivative. <i>Trap</i> : $9 \cdot \frac{4}{3} = 12$; $2 \cdot \frac{1}{2} = 1$.
10(d)	$3x^2 + \sin x$	1 pt rewriting $x^3 - 1 - \cos x$; 2 pts derivative. <i>Trap</i> : the product expands to $x^3 - 1$; the derivative of $-\cos x$ is $+\sin x$.
11	$\frac{3\sqrt{3}}{2} - 1$	3 pts $f'(x) = 3 \cos x - 2 \sin x$; 2 pts value. <i>Trap</i> : $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, $\sin \frac{\pi}{6} = \frac{1}{2}$; swapping them earns 3.
12	$x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$	2 pts $f'(x) = 2 \cos x + 1$; 2 pts $\cos x = -\frac{1}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : stopping after one solution. Both lie in $(0, 2\pi)$; giving only one earns 4.
13(a)	III	u' is a line crossing zero at $x = -\frac{5}{2}$, where u turns, with the sign of u 's slope on each side. <i>Trap</i> : I, the same line with the opposite sign, or a parabola because u is a parabola.
13(b)	IV	v' is a parabola with zeros at $x = \frac{1}{2}$ and $x = \frac{5}{2}$, where v turns. <i>Trap</i> : II, which has the right zeros and the wrong signs.
14	see sketch description	$k' = -2$ for $x < -3$; $k' = 0$ for $-3 < x < 2$; $k' = -1$ for $x > 2$. Open circles at the ends of each segment, at $x = -3$ and $x = 2$. 2 pts per horizontal piece (correct height and interval); -1 once if the open circles are missing or replaced by closed dots. <i>Trap</i> : drawing k 's shape again, or joining the pieces with vertical segments.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 12 + 6 + 12 + 5 + 6 + 6 + 6 = 100$.